**Math 310 Problem Solving for High School Teachers**
**Spring 2016**
**Lecturer: Dr. Paullin**

# MA 310 Class Log

### Wednesday, January 13th

(1) Read through Syllabus.
(2) Assigned Writing 1: Math Autobiography

### Friday, January 15th

(1) Played the Game of 15 and discussed applications to Tic-Tac-Toe.
(2) Assigned Homework 1: Read Chapter 1 in the Zeitz textbook and write up your problem solving process for Game of 100.

### Monday, January 18th

MLK Day - no class

### Wednesday, January 20th

(1) Collected Math Autobiographies.
(2) Played a Word Game and discussed it's isomorphism to Tic-Tac-Toe.
(3) Looked at the Game of 34 Magic Square, and discussed how it's not isomorphic to Tic-Tac-Toe.
(4) Handed out reading: "Is Math a Gift?" by Carol Dweck.
(5) Be prepared for class discussion on readings and Exam 1 on Friday.

### Friday, January 22nd

Snow Day - no class

### Monday, January 25th

(1) Collected Game of 100 Homework.
(2) Class Discussion on the following questions:
  - From Zeitz Chapter 1:
    - From the "Exercises vs. Problems" section ... How often in school have you been faced with problems, rather than exercises? How do you feel about this?
    - What did you think of the mountaineering example of problem solving?
  - From the Dweck Article:
    - Prior to reading this article, how do you view math: as a gift? Or as something that can be developed?
    - What messages do we send in our math classrooms/society? If not ideal, how can we fix this?
  - Can you make any connections between these two readings?
(3) First exam: Isomorphism to Tic-Tac-Toe

**Wednesday, January 27th**

(1) Used time in class to work solutions for 3 problems: Strange Purchase, Hiker, and Penny Piles.
(2) As a group, we started to write up a proof of the Penny Piles problem.
(3) Before next class, read Zeitz Chapter 2, section 1.

**Friday, January 29th**

(1) As a group, we wrote up a more complete proof of the Penny Piles Problem.
(2) We spent a few minutes analyzing the Number Guessing Cards problem.
(3) Before next class, read Zeitz Chapter 2, section 2.

**Monday, February 1st**

(1) In small groups, we used class time to work solutions for 3 problems: Thinking about Thinking-Cards, Tiling with L's, Newspaper Sheet.

**Wednesday, February 3rd**

(1) In larger groups, worked on writing up solutions to either the Tiling with L's problem or the Number Guessing Cards problem.
(2) Homework 2: Write up your solution to one of the above problems.

**Friday, February 5th**

(1) Started class by introducing the topic of Mathematical Induction.
(2) Dr. Paullin used induction to prove $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$
(3) In groups, we worked on the problem, "If n lines are drawn in a plane, and no two lines are parallel, how many regions do they separate the plane into?". We found the answer to be $\frac{n^2+n+2}{2}$
(4) Homework 3 is due Wednesday, February 10th, which is to write up your solution to the Lines in a Plane problem using Induction.

**Monday, February 8th**

(1) We spent class time proving the induction problem, for all $n \geq 4, n! > 2^n$.
(2) Exam 2 on Induction will be Wednesday.

**Wednesday, February 10th**

(1) Exam 2 was postponed with the need for more Induction practice.
(2) We talked some more about proper Induction form.
(3) We practiced the following induction problems:
    (a) Prove $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
    (b) Prove $1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4}$
    (c) Prove $1^4 + 2^4 + 3^4 + ... + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
    (d) Prove $1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$
    (e) Prove $1 + 2 + 4 + 8 + 16 + .... + 2^n = 2^{n+1} - 1$
    (f) Prove $2 + 6 + 12 + 20 + ... + n(n + 1) = \frac{n(n+1)(n+2)}{3}$

**Friday, February 12th**

(1) We practiced the following induction problems:
   (a) The sum of the interior angles in a triangle is 180 degrees, or $\pi$. Using this result and induction, prove that for any $n \geq 3$, the sum of the interior angles in an n-sided polygon is $(n-2)\pi$.
   (b) Show that if $a_i$ and $b_i$ $(i = 1, 2, ..., n)$ are real numbers such that $a_i \leq b_i$ for all $i$, then
$$\sum_{i=1}^{n} a_i \leq \sum_{i=1}^{n} b_i$$
   (You may use the fact that if $a \leq b$ and $c \leq d$ then $a + c \leq b + d$.)

(2) Exam 2 was rescheduled for Monday.

**Monday, February 15th**

(1) Handed out Writing Assignment 2 on the Mathematical Practice Standards of the Common Core, due Friday, February 19th.

(2) Exam 2: Induction

**Wednesday, February 17th**

(1) We generalized the ideas of Permutations and Combinations, without using the formulas.

(2) Be prepared for solving Permutation and Combination problems next class.

**Friday, February 19th**

(1) We spent class time working on the following Permutation and Combination problems.
   (a) There are 5 doors to a lecture room. In how many ways can a student enter the room through a door and leave the room by a different door?
   (b) In how many ways can 3 students be selected from a group of 12 students to represent a school in the inter-school essay competition?
   (c) How many words can be formed by re-arranging the letters of the word PROBLEMS such that P and S occupy the first and last position respectively? (Note: The words thus formed need not be meaningful)
   (d) 8 directors, the vice chairman and the chairman are to be seated around a circular table. If the chairman should sit between a director and the vice chairman, in how many ways can they be seated?
   (e) If Nathan flips a coin 6 times, what is the probability that he gets exactly 3 heads?
   (f) Janice is threading beads on a string. How many patterns can she create if she has 5 purple beads, 3 green beads, and 2 blue beads, assuming she must use all 10 beads?
   (g) An equal number of juniors and seniors are trying out for six spots on the university debating team. If the team must consist of at least four seniors, then how many different possible debating teams can result if five juniors try out?

(h) A bibliophile plans to put a total of seven books on her marble shelf. She can choose these seven books from a mixture of works from Antiquity and works on Post-modernism, of which there are seven each. If the shelf must contain at least four works from Antiquity, and one on Post-Modernism, then how many ways can he select seven books to go on the shelf?

(i) An artist is planning on mixing together any number of different colors from her palette. A mixture results as long as the artist combines at least two colors. If the number of possible mixtures is less than 500, what is the greatest number of colors the artist could have in her palette?

(2) We solved all but the last problem, whose answer is 8. Students who would like to write up a solution as to WHY the solution to the last problem is 8 can receive bonus points. This write up is due Wednesday.

## Monday, February 22nd

(1) We worked the following Combination and Permuation problems in class:
(a) From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

(b) In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

(c) Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

(d) How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated?

(e) A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

(2) On Wednesday, we'll have a group (2-3 people) Exam 3 on Permutations and Combinations.

## Wednesday, February 24th

(1) We practiced some Permutation and Combination problems:
(a) How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of 3 faculty members from the mathematics department and 4 from the computer science department, if there are 9 faculty members of the math department and 11 of the CS department?

(b) The license plates in the state of Utah consist of three letters followed by three single-digit numbers.
    (i) How many different license plates are possible?
    (ii) If Edwards initials are EAM, what is the probability that his license plate will have his initials on it (in any order)?
    (iii) What is the probability that his license plate will have his initials in the correct order?

(2) We took the group Exam 3.

**Friday, February 26th**

(1) Class started with a Special Presentation on Teach for America.

(2) We worked on a few geometry problems, the most interesting of which was:

Imagine that you are on a perfectly smooth sphere as big as the sun. A steel band is stretched tightly around the equator.

One yard of steel is added to this band so that it is raised off the surface of the sphere by the same distance all the way around. Will this lift the band high enough so that you can:

  (a) Slip a playing card under it?
  (b) Slip your hand under it?
  (c) Slip a baseball under it?